Supplementary material 1

When processing geodata, it is assumed that the whole variety of DB attribute relations $x=\{x_i\}\in X$ of space X of dimension R^n is ideally described in parts (layers, fibers) by a continuous system function $F(x)$ using differential geometry stratification (bundle) procedures [1]. For continuous sets (spaces) we realise tangent bundle (X,M,π) over manifolds M based on the ideas of differential geometry [2] π: $X \rightarrow M$, where X is the multidimensional bundle space, M is the bundle base with continuous topology; π is the bundle projection procedure; $Y=\{Y_j\} \rightarrow X$ is fibration space of non-intersecting tangent fibers (layers). In the cartograms, thematic layers Y_j of the fiber bundle space $Y=\{Y_j\}\subset X$ are represented by non-intersecting country contours. The surface of a manifold M is described by differentiable functions $F(x)$ of many variables $x=\{x_i\}$. It is assumed that the system function $F(x)$ is various for systems of different kinds, for which the coordinates x_i, elements ξ and their connections $F(x)$ are understood differently. The continuous function $F(x)$ of a surface M allows us to solve analytic problems by means of differential geometry. Each layer $Y_j(y)$ touches the smooth surface $F(x)$ at the point $x_{0j}=\{x_{0ij}\}$, where $x_0=\{x_{0j}\}$, $y=x-x_{0j}$, $y=\{y_i\}$ is the local coordinate system of the layer (fiber). The set $F(x_0)$ is a discrete base space of the bundle on the set of planes $Y=\{Y_j\}$ tangent at points with coordinates $x_0=\{x_{0j}\}$ - elements of the $b_j\in B$ bundle base $B=F(x_0)$. In the neighborhood of the points $F(x_0)$ and $x_{0j}=\{x_{0ij}\}$, the function $F(x)$ is approximated in the fiber with high accuracy by the equations [3]:

$$F(x) = f(y) + F(x_0), \tag{1}$$

$$f(y) = a \cdot y = \sum_{i=1}^{n} a_i y_i = \sum_{i=1}^{n} \frac{\partial f}{\partial y_i} y_i,\ a_i = \frac{\partial f}{\partial y_i},\ f(y) = F(x) - F(x_0) \tag{2}$$

The first equation reflects the superposition of the geographical environment characteristics $F(x_0)$ on the universal integrating bilinear function $f(y)$ of relative variables y, describing the deviations of the local state of the system (in the fiber) under the influence of internal forces $f(y)=a{\cdot}y$ (scalar product of vectors $y=\{y_i\}$ and $a=\{a_i\}$, a_i – sensitivity coefficients of changes in the functions $F(x)$ and $f(y)$ at a unit change of variables x_i and y_i). Regression and factor analysis methods are used in statistical evaluation of the coefficients a_i of the formula $f(y)=a{\cdot}y$ (2). The universal meta-analytical function $f(y)$, is the same for all environmental conditions, which allows in local coordinates $y=\{y_i\}$ to reduce different dependencies $F(x)$ to one typical relation $f(y)$, to compare them by comparing (identifying) different tangent points ($F(x_0)$, x_0) characterising the environment.

The procedures of fiber bundle over manifolds make it possible to pass from a continuous space of characteristics $x=\{x_i\}\in X$ to a discrete set of fibers $Y_j(y)$ corresponding to isolated states (types) of systems indexed by a representative of type b_j with the parameters of the centre of the layer $x_{0j}=\{x_{0ij}\}$. In a general form, this approach is realised in H. Everett's concept [4] of the plurality of worlds: there are many parallel and equal copies $Y_j(y)$ of reality in the form of a large number of possible states. In the process of measurement, it is stratified into alternative projections of observation, where independent branches of a single F-function describing formally non-interacting worlds, in this case – countries, are allowed. All branches of the F-function independently exist and develop along a path different from other branches.

The superposition $G[f(y)]$ of functions of layers $f(y)$ gives rise to a space of principal fibrations where the layers (fibers) are not linear spaces $f(y)$ but instances of $G[f(y)]$ of the group, e.g., by the multiplication operation of functions of the form $G[f(y)]=\exp[Af(y)]$. Multiple superposition $G\{G[f(y)]\}=\exp\{B\exp[Af(y)]\}$ is allowed, giving the functions a hierarchical form [5]; A, B are real or complex constants. The direct and inverse transformation $G\{G[f(y)]\} \leftrightarrow G[f(y)] \leftrightarrow f(y)$ is allowed, bringing complex functions to the linear form (2).

References

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Supplementary material 2

Reliability is the property of an object to preserve in time within the established limits the values of all parameters that ensure the fulfilment of certain functions under given conditions [1]. Reliability calculation methods are primarily developed in technical sciences [2]. Reliability is associated with the absence of failures (losses, casualties, accidents) in operation (activity) – the property of an object to preserve its operability (vitality) in time or in the process of working on the accepted parameter (age, distance, number of applications). Failure in the epidemic process is associated with cases of infection, loss of health or death of a person. Reliability characterizes the situation in relative terms, without taking into account the number of objects in the system, in their expected average behavior.

The development of the epidemic is reflected in reliability indicators, for example, by the decreasing reliability function $P^*(t)$ – the probability of failure-free operation of the organism, preservation of health by the moment t in days from the beginning of the epidemic. The cumulative probability function of failure accumulation (unreliability) is $F^*(t)=1 - P^*(t)$, so that in the epidemiological SIR model: $S(t)=S_0P^*(t)$, $I(t)=S_0F^*(t)$. Based on the reliability function $P^*(t)$, other indicators are calculated:

$$P(t) = -\frac{dP^*(t)}{dt},\ E(t) = -\ln P^*(t),\ E^*(t) = -\ln F^*(t),\quad p(t) = \frac{dE}{dt} = \frac{P(t)}{P^*(t)}, \tag{3}$$

where $P(t)$ is the probability density function (distribution) of failures, in particular, the proportion of the number of confirmed infections per unit of time (day); $E(t)$ is integrated safety (possibility of not getting sick by time t); $E^*(t)$ is integrated hazard (threat, possibility of getting sick by time t); $p(t)$ is the intensity of failures, differentiated hazard, or risk, equal to the proportion of those who fall ill per unit of time in the sensitive population $S_0(t)$ (morbidity).

To describe the epidemic process $F^*(t)$, the twice exponential Gompertz-Gumbel' distribution function is used [3-4], which is a special case of the Fisher-Tippett-Gnedenko distribution for extreme events [5]. Three types of distributions of extreme - largest or smallest - values in a large sample are distinguished: Gumbel, Fréchet, and Weibull probability functions. These and other functions form the basis of mathematical support for approximating the statistical dependence of the proportion of infected $F^*(t)$ and uninfected $P^*(t)$ of the sensitive population of the territory on the time t of the events (the moment of disease detection). From among these functions, the best distribution law is selected for displaying statistical plots. The search is oriented on revealing the natural law of the true distribution, which agrees by statistical criteria with the available data for an individual country not only for the cumulative infection curve $F^*(t)$, but, more importantly, with the indicators of distribution density $P(t)$, intensity $p(t)$ and integrated hazard $E^*(t)$ of infection.

For the largest and smallest values, the distribution of Gumbel's law follows a specific form:

$$F^*(z)=\exp[-\exp(-z)], \quad (4)$$

$$F^*(z)=1-\exp[-\exp(z)],\ P^*(z)=\exp[-\exp(z)], \quad (5)$$

where $z=\alpha x+\beta=\alpha(x-x_0) \sim \alpha(t-t_m)$, x_0, t_m are coefficients of locality (localization) and $\lambda= 1/\alpha$ – scale (dimensionality). In this case, the transition from (4) to (5) is associated with the replacement of the independent variable $z \rightarrow -z$ or $x \leftrightarrow x_0$. The distribution density functions for variants (4) and (5) have the form (Figure 3):

$$P(z)=\exp(-z)\exp[-\exp(-z)],\ P(z)=\exp(z)\exp[-\exp(z)]. \quad (6)$$

In both cases, the maximum P_m of the function $P(z)$ is observed at z=0 and is equal to $P_m=1/e$. For $z=\alpha(x-x_0)$ the maximum of $P_m(x,x_0)=\alpha\exp(-1)$ takes place at $x=x_0$, which allows us to determine the coefficient $\alpha=eP_m$ approximately by the position of the extremum (x_0, P_m) of the statistical plot $P(x,x_0)$. Similarly for $z=\alpha(t-t_m)$ is $\alpha=eP_m=eP(t_m)$.

The Fréchet distributions $F^*(z)=\exp(-z^{-\alpha})$, $z>0$, $\alpha>0$ and the Weibull $F^*(z)=\exp(-(-z)^{-\alpha})$, $z\leq 0$ or $z>0$, $\alpha>0$ are reduced to equations of the form (4)-(5) in new proper time variables $\tau=\theta\ln(t/t_m)+t_m$ ($1/\theta$ is the stretching constant of the graphs) when the variables $z^{-\alpha}=\pm\exp[-\alpha(\tau-\tau_m)]$ are replaced:

$$F^*(z)=\exp(-z^{-\alpha}) = \exp\{-\exp[-\alpha(\tau-\tau_m)]\}. \quad (7)$$

The value of the coefficient α is found similarly by the position $\tau=\tau_m=\theta\ln(t_m/t_m)+t_m= t_m$, $z=\pm 1$ of the maximum P_m of the distribution density function $\alpha=eP_m$, $\tau(t_m)=t_m$.

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